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The Fiber Bundle at the Gates of Metaphysics. Challenging Maudlin's Proposal

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Abstract In a recent book (*The Metaphysics within Physics*), Tim Maudlin reconstructs metaphysics by taking inspiration from the gauge theories interpreted in the fiber bundle framework. I call his project the "fiber bundle metaphysics". Primarily targeted not to Humean Supervenience, but to any metaphysics employing the relation of resemblance among objects (D. Lewis, D. Armstrong), Maudlin's project is novel and promising. I critically analyze the arguments by identifying several objections stemming first from metaphysics. The metaphysician questions whether gauge theory represented through fiber bundles is apt to reform metaphysics. It needs, I claim, a firmer commitment to realism. Second, she cannot see how Maudlin accommodates the metaphysical "loneliness" of objects in the fiber bundle metaphysics and complains that the mathematical structures of the fiber bundle metaphysics are weakly discernible only. A second class of objections stems from the physics of gauge theories. I see a "conventional" solution to Maudlin's path-dependency argument against Lewis's "pure metaphysical relations": other invariants of affine connections can play the role of internal properties and relations. I raise an objection and address it regarding the duality of the fiber bundle representation which is deeply divided among two types of bundles, corresponding to different ontologies: gauge fields and spacetime diffeomorphism. Several possible paths towards more realistic interpretations of the fiber bundle are briefly discussed. Finally, I bring in the provlem of locality, separability and I emphasize some criticisms. My conclusion is that Mauldin's project is assuring, but not powerful enough to reform metaphysics.

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1 Introduction

The new analytic metaphysics that has resurged in the last decades (D. Armstrong, D. Lewis, Th. Sider, *i.a.*) needs to confront a new philosophy of science (which outgrew its logical positivist roots) and to face nowadays' sciences. Can the new metaphysics peacefully coexist with sciences which are radically different from those of the Quine and Carnap days?

Philosophers have contemplated the relation between science and metaphysics and some gave a harsh verdict: metaphysics should learn from its past mistakes and let be reformed by sciences! Witness the strong opposition to speculative metaphysics (including the new "analytic metaphysics"), frequently dubbed the "new scientism"[1]:

[1] ***Scientism:*** *In the light of contemporary theories in physics, metaphysicians need to reform their fundamental ontology.*

Several doctrines in metaphysics: universals, dispositions, modality, *haecceity*, *quiddity*, part-whole and composition relations, and, last but not least, Humean Supervenience were all under scrutiny by philosophers adopting [1]. We know that metaphysicians belong to a conservative crowd: many clench on a pre-Einsteinian concept of time, to pre-Bell concepts of locality and separability or to some medieval concepts of objects and their boundaries. Ditto about universals: many metaphysicians quixotically stick to an abstract concept of "something" (the "universals") that helps in grasping resemblances and differences among objects. For the new scientists, natural sciences and physics in particular, are more efficient in producing evidences about the ontology of the world than millennia of metaphysics (Maudlin 2007, 78). Quantum mechanics and its interpretations, quantum field theory, special and general relativity, cosmology, genetics, all can be used to knock out pieces of standard metaphysics.

In the line of [1], T. Maudlin has recently surveyed some doctrines in analytic metaphysics: Humean Supervenience, Theories of Universals, etc., and argued against them (Maudlin 2007). Humean Supervenience is deemed both too restrictive and too permissive.[2] In arguing that it is too permissive, Maudlin adopted two positions:

[2] ***Eliminativism****: Repeatable, qualitatively identical local properties, both intrinsic and extrinsic, employed in the Humean ontology, are not compatible with contemporary physics (esp. Quantum Mechanics).*[3]

[3] ***No Resemblance****: The relation of "resemblance" among objects is not well-defined when situated in the context of contemporary physics. A new approach is needed.*

Maudlin seeks adequate alternatives to some doctrines in metaphysics by looking outward: in the spirit of [1], the most promising replacement comes from contemporary physics:

> the primary object of this [chapter] is to sketch enough of that theory to make clear how it constitutes an alternative to the theory of universals, and to the theory of tropes, and to all of the various kindred metaphysical hypotheses that are considered in the philosophical literature. (Maudlin 2007, 80)

[1] I do not use this term in Putnam's pejorative sense, but as discussed recently by P. Maddy, J. Ladyman et al., etc.; see (Putnam 1983),(Maddy 2007),(Ladyman et al 2007).

[2] It is "restrictive" because Maudlin advocates a form of primitivism in respect of laws of nature and the direction of time.They need, Maudlin argues, to be added to the Humean Mosaic. This argument does not concern us here.

[3] The literature on this topic is vast. See (Maudlin 1994), but also D. Mermin, P. Teller etc.

Enter gauge theories, interpreted in the fiber bundle framework.[4]

Why gauge theories and why in [3]? Are they part of the crew of exemplar scientific theories able to teach metaphysicians a lesson? In fact they do, but in a different context. They constitute a subject *per se* in philosophy of science, rich in metaphysical consequences: the gauge argument is typically used to show that separability and locality, assumptions strongly related to Humean Supervenience, are violated by gauge fields (Healey 2004).[5] Another revision to standard metaphysics is usually inferred from gauge argument: the "metaphysical holism", opposed to Humean Supervenience.

As Maudlin employs gauge theories to argue for [3], his approach goes beyond separability, locality and alike, and purportedly extends further than Humean Supervenience. I deem [3] more general and novel because several major currents in contemporary metaphysics: D. Lewis' Humean Supervenience, D. Armstrong's Theory of Universals (essentially, a non-Humean doctrine) as well as Trope Theory and Theory of Primitive Naturalness (a set theory at base) are under the harsh scrutiny of this new flavor of scientism. Here are some consequences of Maudlin's argument to be discussed in this paper:

[4] *Spacetime and physical interactions do matter in building metaphysical systems.*

[5] *As an answer to [3], "fiber bundle metaphysics" constitutes the viable alternative to the metaphysical relation of resemblance.*

[6] *"Metaphysicians need to speak the language of fiber bundle".*

I agree with Maudlin's broad project [4]. But when it comes to the constructive part [5] and to the slogan [6], I am more distrustful and argue that we need to be more nuanced when learn lessons in metaphysics from gauge theories.

2 Two Negative Arguments

D. Lewis and D. Armstrong both defended a form of metaphysical realism about properties and/or universals. Language, commonsense thought and science sort, classify and categorize objects based on resemblances and differences among objects. Lewis adopts the traditional realism and emphasizes the importance of properties to account for objective sameness and difference which are "joints in the world, discriminatory classifications not of our own making" (Lewis 1999, 67). Indeed, it would be tricky to account for resemblance without properties or relations. Without resemblance, one may conclude that "the only real structure of the universe is its cardinality" (Maudlin 2007, 85). It seems that all parties involved want to avoid a strong nominalism à la Quine.

[4] To serve my future argument, I avoid calling fiber bundle a "theory" . Fiber bundles are discussed in the philosophical literature: (Auyang 1995),(Healey 2001),(Penrose 2005),(Healey 2007), more technical presentations are (Baez and Muniain 1994),(Fatibene and Francaviglia 2003).

[5] See (Auyang 2001),(Leeds 1999),(Teller 1997) and the most comprehensive approach to gauge theory available (Healey 2007). Metaphysical holism is described in (Leeds 1999), (Healey 2007, sect. 4.5).

2.1 Sparse properties in Lewis

What is needed to account for resemblance? After recanting the Quinean position, Lewis accepted after 1983 that *sparse* properties, a type of "natural" properties, play the central role in marking out the genuine qualitative similarities and differences in objects (Lewis 1983). Ditto about relations: gerrymandered and artificial relations need to be differentiated from *sparse* and natural relations. Sparse properties, which Maudlin is interested in, "carve at the joint throngs", they are highly specific, and are only just enough to characterize things (Lewis 1986, 60). For both Armstrong and Lewis, the world is a collection of spatio-temporal points instantiating the localized intrinsic properties and external spacetime relations among them.

In Lewis, "metaphysical pure" properties and relations are defined for lonely objects which exist by themselves in possible worlds. A relation is metaphysically pure when "it is at least possible that the relation be instantiated in a world in which only the *relata* of the relation exist" (Maudlin 2007, 86-7).

Maudlin's negative argument is twofolded: (a) there are no metaphysically pure external relations, and (b) that there are no metaphysically pure internal relations. Consequently, Lewis' theory is not able to compare two objects by comparing the universals instantiated by the two objects.

2.2 No metaphysically pure external relations

The argument for (a) is based on the central role spacetime plays as an external relation among objects, the spatio-temporal relation, understood as the "distance" between two objects, being the best candidate (Maudlin 2007, 87-89):

[**7**] *Identify the external relation between* O_1 *and* O_2 *with the spatial distance between* O_1 *and* O_2*.*

It is easy to see that "distance" depends on "length" because two objects cannot be, let us say two meters apart, in a world where only those two objects exist. Length also hides a universal quantification over all possible continuous paths in spacetime. In Maudlin's ontology, we need paths in spacetime even if they are not occupied by real objects.

2.3 No metaphysically pure internal relations

In respect of (b), Maudlin takes the direction of a vector as a *prima facie* candidate for an internal property of a spacetime point:

[**8**] *Identify internal relations with vectors (directions)*

In Euclidean space, two vectors can point in the same direction even if they are located at different points in space. Two vectors are either parallel or not, no matter where they are located. This is, I take, a fortunate marriage between metaphysics and classical physics. But for other philosophers, even this relationship is not intrinsic as vectors point toward regions of space in its *neighborhood*. In other words, as J. Butterfield showed, one cannot be "pointillist" and believe in intrinsic properties associated

to a point—even in classical mechanics (Butterfield 2006).[6] But let us grant for the sake of the argument that classical mechanics can be wedded to internal properties of spacetime points.

Maudlin argues that this jolly marriage does not take place in non-Euclidean geometry. Without a connection, vectors, directions, lengths etc. cannot be transported from one place to another and consequently cannot be compared. They live in isolated tangent spaces attached to each point. The notion of an *absolute* comparison of directions of vectors has to be abandoned.[7] We can compare a direction $\overrightarrow{V_1}$ at a point A_1 to another direction $\overrightarrow{V_2}$ at another point A_2 *relatively* to a pathγ.

If we adopt [8], then two directions cannot be compared from one location to another unless there is a third entity, the spacetime, and it is equipped with a connection. Parallelism and the connection are needed, so a relation defined by [8] is not metaphysically pure (Maudlin 2007, 93). Notwithstanding its *purity*, a relation can be defined for all practical purposes, albeit conventional.[8]

Without further ado, one can see that if metaphysically pure properties depend on metaphysical pure relations, there are no metaphysically pure properties either. Here are Maudlin's main conclusions of his main negative arguments:

[**9**] *There are no metaphysically pure external relations between two objects (Maudlin 2007, 87-89)*

[**10**] *There are no metaphysically pure internal relations (Maudlin 2007, 89-93):*

[**11**] *There are no metaphysically pure relations (and no metaphysical pure properties).*

[**12**] *Negative consequence: Analytic metaphysics cannot explain resemblance and dissimilarity among objects.*

To exemplify [12], take a toy model in which quarks have three possible color charges: "red" "blue" and "green". Calling a quark "red" does not mean it bears a *similarity* relation of color to other red quarks. In the same vein, whether two quarks have the same color depends on the path in spacetime we choose to relate the two points associated with the quarks. Maudlin rejects that there is:

> any metaphysically pure relation of comparison between quarks at different points, since the only comparisons available are necessarily dependent on the existence of a continuous path in space-time connecting the points. [...] (Maudlin 2007, 96).

And if the metaphysician believes that physics provides universals, then she's wrongheaded: "physics is telling us there are no such things" (*idem*). This does not seem a promising path to follow in metaphysics...

[6] See the rich discussion in Butterfield of the pros and cons of pointillism in classical mechanics.

[7] A parallel transport is closely related to the connection and implicitly to its covariant derivative. Also, intuitively, the parallel transport of a vector along a closed loop on a sphere is proportional to the area of the loop.

[8] "But in a practical sense, comparisons can be made between directions at different points so long as parallel transport along the paths that anyone is likely to use gives results which are, within common margins of error, the same [...] the truth conditions for claims about parallelism even where there is, strictly speaking, no absolute, metaphysically pure relation of 'pointing in the same direction' at all." (Maudlin 2007, 93)

3 The Positive Project: Fiber Bundle Metaphysics

Is this the end of the story? Not at all! There is a positive and constructive part of Maudlin's project. He claims that fiber bundle is enlightening and can be used to reconceptualize and reconstruct metaphysics because it is able to explain similarity without using metaphysically pure relations. I call this project the "fiber-bundle metaphysics" and I fathom it as a positive argument overall. I focus now on [5].

One can see fiber bundle as a natural step in the progress of physics. Classical mechanics describes some quantities as scalars: mass, electrical charge, density, etc. In differential geometry we want to know how quantities associated to a field transform between coordinate systems. Scalars and metaphysics of properties and/or universals would work hand in glove. This may sound trite, but some metaphysicians need to be reminded that a physics of scalar fields lies about the world! Classical Mechanics needs more than scalars to represent forces, velocities, accelerations, etc., special relativity needs 4-vectors, and general relativity needs tensors and other more sophisticated objects. Why should we stick with scalars when reality is richer and more complicated? An identification such as [8] is far too restrictive and do not mesh well with the theoretical physics.

Even classical electromagnetism, one of the simplest field theory, cannot be represented by a vector *space* and a vector *field* is needed:

[13] *Identify internal relations with vector fields*

There is a tremendous consequence of this assumption. Even electromagnetic fields cannot be differentiated anymore in the usual way because vector fields cannot be compared at two locations—an operation needed in the naïve differentiation. Maudlin's negative argument can be subsumed to slogan in differential geometry: "differentiating vectors is not a trivial affair" (Baez and Muniain 1994, 223). The derivative of a function:

$$\lim_{\varepsilon \to 0} \frac{f(x+\varepsilon) - f(x)}{\varepsilon} \tag{1}$$

is not well-defined even in gauge theories and in General Relativity. In the long run, approaching physics by partial differential equations are seriously questioned here and the possible alternative is the "connection" on a fiber bundle. 1 is not a viable option for gauge theories.

If Maudlin is right about the fiber bundle framework, it constitutes the best framework to unitarily describe affine connections, gauge theories and spacetime manifolds. Some enthusiasts think of fiber bundle as a "natural geometric concept" and identify gauge fields with fiber bundles: "Fiber bundle provides a kind of unified framework for us to compare the conceptual structures of various major theories [general relativity, gauge theories, classical mechanics]" (Auyang 1995, 130).[9] Irrespective of these slogans, fiber bundle is *prima facie* nothing else than a quadruple of several mathematical structures:

$$\mathbf{B} = (E, M, \pi; F) \tag{2}$$

where E is the "total space", M is the "base space", F is a standard fiber (all are topological spaces) and $\pi : \mathrm{E} \to \mathrm{M}$ is a continuous surjection; for details see (Healey

[9] For Wu and Yang gauge fields are based on geometry: "To us it is remarkable that a geometrical concept formulated without reference to physics should turn out to be the basis of one, and indeed maybe all, of the fundamental interactions of the physical world". (Wu and Yang 1975, 3856).

2007, 7-13) and a more detailed description in (Fatibene and Francaviglia 2003, 12,34, 80sqq.). The map π is called the projection map (or bundle projection) and satisfies a *local triviality* condition: the base space M can be covered by open covering $\{\mathrm{U}_i\}$ such that for any i there will be a diffeomorphism:

$$\mathrm{t}_i{:}\pi^{-1}(\mathrm{U}_i) \to \mathrm{U}_i \times \mathrm{F} \tag{3}$$

The trivialization is *local* because for different U_i the diffeomorphism t_i is not the same in general. One can think that the most trivial way of representing the manifold M and the fiber F is by simply taking their Cartesian product: the trivial bundle is then

$$(M \times F, M, pr_1; F) \tag{4}$$

where pr_1 is the trivial projection map:

$$\mathrm{pr}_1\colon \mathrm{M} \times F \to M \tag{5}$$

When the fiber F is a vector space and the transition maps t_{ij} are linear isomorphisms, the internal space at a point m of M is a canonical vector space. A bundle with a vector space is called a vector bundle. That roughly would correspond to the situation depicted by [8].

If the fiber F is a Lie group, we call the fiber bundle a *principal bundle*

$$\mathbf{P} = (\mathrm{P}, \mathrm{M}, \pi; \mathrm{G}) \tag{6}$$

Principal bundles do not have a preferred element like the zero element (in the vector bundle or the affine bundle) and do not allow always global sections (Fatibene and Francaviglia 2003, 23-29,34).

To bolster the positive projects [5] and [6], Maudlin argues that fiber bundle "provides new mathematical structures for representing physical states, and hence a new way to understand physical ontology." (Maudlin 2007, 101) and because "the structure of the physical world might be that of a fiber bundle", [...] if fiber bundle does not "translate well into the old categories, so much the worse for the philosophical debates". Finally, we have at least some reasons to think that "fiber bundle theories, and not the theory of universals or its kindred, are true" (Maudlin 2007, 103). In the following section I argue that several steps are missing in Maudlin's argument and that his realism commitment to the truth of the fiber bundle theories is too curtailed.

4 Two metaphysical rejoinders

I devise some objections to Maudlin's project: the first three are metaphysical in nature; the last two are situated in philosophy of physics and relate mathematics, physics and metaphysics in the context of fiber bundle framework. More general morals are drawn in the concluding section.

4.1 Metaphysical rejoinder (1): Representation and Hypostatization

Metaphysicians advocate the independence of metaphysics from science and their "division of labor": metaphysics begins where science ends and *vice versa* (Conee and Sider 2005). Several reasons are invoked: first, metaphysics is a perennial reflection, independent of empirical and scientific results; second, unlike scientists, metaphysicians are able to marshal strong and plausible commonsense intuitions or the ordinary language to support their view. I disagree with both reasons, but they are germane to the present debate. In lieu of independence, I see constructive coexistence and compatibility between metaphysics and science in the spirit of [1].

Lewis was both a scientific realist and a metaphysical realist and believed in both the *correctness* and *completeness* of the scientific worldview: "It is the task of physics to provide an inventory of all the fundamental properties and relations that occur in the world [...] we may reasonably think that present-day physics already goes a long way toward a complete and correct inventory" (Lewis 1999, 292).

In the line of Lewis's realism, in order to reform metaphysics, philosophers who embrace scientism [1] need to commit to theories which have more content than mere representations. Maudlin is not analytically clear on this aspect. The metaphysician asks whether our scientific theories are complete enough and true enough to reform or displace metaphysics:

Q: 1 *Which scientific theory and which of its components are appropriate to reform metaphysics?*

A quick metaphysical rejoinder to Maudlin is drawn from the pessimistic meta-induction: are our current scientific theories, to be displaced and refuted sooner or later, able to ground metaphysical systems? Many scientific theories are open to interpretations, saturated with representational surplus, and many do not mesh well with other well-establish theories. For example, Lewis was unenthusiastic to take lessons from quantum mechanics because its instrumentalist frivolity, doublethink logic and "supernatural tales about the power of the observant mind to make things jump" (Lewis 1986, xi). Trivially enough, history of science can bolster this argument. Although ether, phlogiston, action, energy, phase space, vacuum, etc. all once partook in the central principles of physics, philosophers have not reformed metaphysics to accommodate them. Analytic mechanics of the 19^{th} century represented the physical world in configuration or phase spaces. We could interpret classical mechanics in the phase space framework as saying something about a 3-N world. Is the wannabe 3-N revisionism of metaphysics trustworthy? Not at all—because we do not live in such a space! The 3-N formalism was only a "map" that helped us in representing systems of N particles. The metaphysician asks Maudlin the same question: are we going to revise the theory of universals based on the fiber bundle framework *without any further ado*?

I see roughly two scenarios for the fiber bundle metaphysics: (i) it is in the same boat as the "phase/configuration space" metaphysics, or (ii) it will have a fate akin to that of the metaphysics of spacetime which successfully reformed the metaphysics of time in the early 20^{th} century. Taken at its face value, scientism [1] implies that our theories in modern physics have or will have the deep impact on metaphysics. The reformers blithely think that we're in cases like (ii). I claim that Maudlin's fiber bundle metaphysics is closer to the (i).

This argument links metaphysics to scientific representation and scientific language. Both metaphysics and philosophy of science had tumultuous relationship with philos-

ophy of language. Maudlin criticizes metaphysics for being trapped in our limited conceptual scheme inspired by ordinary experience and language.[10] Should we be realist about universals, possible worlds, etc.? No, as Maudlin argues, because we commit the sin of hypostatization. Here the metaphysician can strike back with the same weapon. Fiber bundle is nothing more than a mathematical representation and mathematics is another type of language. We have lot of arguments against reification of mathematical entities: fictionalism, weaseling out mathematical entities, nominalism, etc. Without further caveats, the fiber bundle metaphysics errs exactly where Aristotle did: it takes the language of the fiber bundle formalism for the structure of the world. This does not differ from taking too seriously similarity, resemblance, intrinsic properties.

One more suggestion from the metaphysics for Maudlin's project: Kripke once said that we should not believe that mathematical formalism can "grind out philosophical results in a manner beyond the capacity of ordinary philosophical reasoning. There is no mathematical substitute for philosophy" (Kripke 1976, 416). Lewis and Kripke insinuate that neither the interpretation of a scientific theory, nor sheer mathematical formalism is enough to displace doctrines in metaphysics. The skeptic metaphysician would answer Q-1 in the negative: no scientific theory is able to reform metaphysics and neither do interpretations, formalism or models. As the metaphysician's argument goes, changing metaphysics it is an internal affair.

There is another way to answer Q-1: we can reform metaphysics only if we have a commitment to the entities of the scientific theory. The metaphysician expects more from the scientific theory ready to reform metaphysics. This is a question that the fiber bundle metaphysics needs to address:

Q: 2 *What is the fiber bundle? Interpretation, theory, model or representation?*

My answer to Q-2 is the latter: fiber bundle is a mathematical representation or a mathematical model powerful enough in the context of gauge theories and arguably in the context of general relativity, albeit some philosophers of science look at it with a jaundiced eye. Fiber bundle qualifies neither as a physical theory, nor as an interpretation of a physical theory. Maudlin's project and projects of its ilk need to be supplemented with a clearer commitment to a form of scientific realism or with an indispensability argument about fiber bundle.

Albeit convenient, a mathematical representation cannot drive the physical reasoning. To paraphrase Kripke, a representation, no matter how powerful, teaches us no lessons in metaphysics. Representations bring in surplus structures and supplementary work is needed on the behalf of philosophers. In the line of the pessimistic argument above, very powerful representational tools were around all the time in science. Why should we build a metaphysical system based on representations?

The lack or, on the contrary, the abundance of interpretations, can hinder the reformist. A Lewisian insists that a scientific theory is not ready to reform metaphysics as long as the theory itself is open to inconsistent interpretations. Conflicting interpretations can produce trouble and one interpretation needs to be cherry-picked by

[10] Most notably, Aristotle fell prey to an error of mistaking the grammatical form of the language for ontological structure. Russell noted that "the English language, as now used by philosophers, offends by provoking erroneous metaphysical beliefs. Syntax induces misleading opinions concerning the *structure* of the world (notably in the attribution of ontological significance to the subject-predicate form), while vocabulary, by promoting the hypostatization of pseudo entities, encourages false beliefs concerning the *contents* of the world." (Russell 1921, 192).

the metaphysician. But the absence of an interpretation is even more damaging: the question is whether the "fiber bundle metaphysics" entertain a specific interpretation.

4.2 Metaphysical rejoinder (2): Objective Loneliness and Indiscernibility

The following two challenges are more specific related to the fiber bundle metaphysics and they echo some of metaphysician's frustrations.

First, it is the definition of metaphysically pure relations: exemplification or instantiation of intrinsic properties is *by definition* independent of the presence of objects and instantiation of other properties. Intrinsic properties do not depend on the presence of a spacetime manifold or the presence of physical fields. Fiber bundle metaphysics restricts too much the context in which intrinsic properties could have been defined. The metaphysician could not dovetail Lewis' "lonely" objects and the fiber bundle metaphysics. If we adopt [7], spacetime comes already equipped with a connection and potentially with a curvature which signals the presence of matter fields. But the object is not alone as such. If in the line of [1], Maudlin preaches the slogan "loneliness is unphysical!", then so much the worse for the metaphysician! This takes us back to square one: accept the division of labor cherished by some metaphysicians, acknowledge the limits of physics and reject any external reform of metaphysics. It is very likely that the debate is purely verbal and that metaphysicians and Maudlin talk past each other.

Last but not least, the discernibility of the mathematical structures troubles the metaphysician. Two vectors pointing in the same direction are *indiscernible*, but *different* geometric objects. Some suggested that in vector space the "thisness" can help individuate vectors (Adams 1979),(Auyang 1995),(Weyl 1949). Fiber bundle suffers from the same lack of discernibility. There are problems with counting the possible fiber bundles at a spacetime point and then with differentiating distinct possible connections between points. The various representations of the connection are underdetermined. Distinguishing geometrical objects is much trickier than distinguishing numerical (arithmetic) objects, which is manifest even in the case of vectors. For gauge fields, the situation is disastrous: by "gauge freedom" multiple representations of a single geometrical object are available. Gauge theories are blatantly ambiguous: the symmetry of the theory prevents us to pick the real distribution of gauge phases (Healey 2008). Metaphysicians are not happy to give it away because discernibility plays a central role in the theory of identity. Although it is a virtue of Maudlin's argument that it is impossible to identify one geometrical structure at point P with another one at point P', what metaphysician could accept such a lax definition of identity? Do we need to reform the theory of identity together with the resemblance relation? As G. Catren remarked, an active transformation of the internal space can be converted into a passive transformation of the coordinate system in the base space, so the field's internal state does not have a gauge invariant meaning (Catren 2008).

An immediate warrant to Maudlin's argument [5] is that we need to give up concepts such as loneliness in possible worlds, discernibility and even identity in contemporary metaphysics. Or, alternatively, what fiber bundle metaphysics suggests is perhaps weaker concepts of loneliness, discirnibility and identity in which spacetime and the presence of matter (gauge) fields play the key role.

5 Three objections and three responses from physics

5.1 Conventionality and invariance in the connection

My last objections are related to the physics of the fiber bundle framework. First, one can assuage Maudlin's negative claims [9] and [10] by insisting on *local* invariants as "physical correlates" of internal relations:

[14] *Identify internal relations with the local invariants of the connection.*

Although physical space is not invariant under Euclidean transformations, it is most probably invariant under *affine* transformations. If we transport universals from one place to another through a spacetime with a connection, we'd better find its invariants. Affine connections do not preserve angles, shapes, lengths, areas, etc. but do preserve *colinearity* of points. Why not identifying colinearity of points in the tangent space with internal properties? In regard to [14], the scientist reformer demurs in two ways: what if physical space is not described by an affine connection? Perhaps tomorrow we'll discover that the projective or the "schme-jective" connection is closer to the physical reality. The short reply to this objection is that even in the most general cases of connections, there are some quantities that are invariants. Second, the reformer can ask: what if spacetime is not equipped with geodesics, has non-unique geodesics or lack them locally? Albeit possible, such spacetimes are unphysical and consequently of little interest here.

Second, if [14] is not appealing, we can use *geodesics* as preferred paths to define resemblance. It is a theorem in differential geometry that in a spacetime equipped with an affine connection geodesics exist *locally* and are *locally* unique. We can always use geodesics to transport vector spaces from one place to the other. In this case, Maudlin's argument [9]-[12] remains sound, but it becomes trivial as the metaphysician can espouse a conventional definition of resemblance:

[15] *Identify internal relations with vector spaces and transport them along geodesics.*

A more general question is whether we always use identifications such as [15] to find physical correlates of internal relations or properties. This is not trivial and more work is needed. But Maudlin's argument is incomplete, at best. I claim that there are ways to reply to Maudlin's argument [9]-[12] which are not, I assume, superficially jerry-rigged, as Maudlin suspected (Maudlin 2007, 101).

Here are some morals stemming from this reconstruction. Fiber bundle metaphysics does not force the metaphysician giving up her theory of internal property, but warns her of: (a) the element of conventionality in the definition of resemblance, (b) the role of spacetime structures in analytic metaphysics and, last but not least, (c) the distinction local-global plays in metaphysics.

5.2 The double life of fiber bundles

Fiber bundle is deemed a reliable tool to represent gauge theories under one representation and hence its distinctive unificatory power. By choosing different vector bundles we represent unitarily several theories which looked differently in their Lagrangian, Hamiltonian or partial differential equation formulations. Does it leave beside gravitation? There is a couple of good news: in General Relativity the vector bundle is the

tangent bundle of spacetime and there is a striking similarity between the role gauge potentials play in the fiber bundle space and the role of the affine connection in general relativity. The fiber bundle framework carries a very strong flavor of geometrization, in the sense that it represents parts of a physical formalism by the means of geometrical structures and seemingly extends the geometrization from general relativity to gauge theories (Cao 1998).

Unfortunately, there is a "glitz" east to succumb to: as J. Earman worried, fiber bundles are excessively powerful and flexible: "[...] we can see fiber bundles all over the place. What is needed is an explanation of what the relevant fiber bundle structure is and how it arises" as the majority of physical theories "don't wear the fiber bundle structure on their sleeves" (Earman 2002, 202). One consequence is that the fiber bundle metaphysics is "too good to be true", as in gauge theories, we can't separate the facts for mere fictional representations (Martin 2002). The other bad news is that gravitation does not fit seamlessly the fiber bundle framework.

Bluntly put there are two types of fiber bundle: (A) The natural bundles which are suitable to describe General Relativity and any general covariant theories; (B) gauge natural bundles suitable to describe gauge theories (Kolář et al 1993). This division makes the framework dualistic in treating natural bundles and gauge bundles on different assumptions. What happens with the natural fiber bundle once one adopts the gauge bundle and *vice-versa*? Several authors showed that the diffeomorphism group of gravity Diff($\mathbb{R}^4$) is not the gauge group of any fiber bundle (Weinstein 1999). While all Yang-Mills fields are compatible with a flat (Minkowskian) spacetime background (and likewise with a curved background spacetime), gravitation is not. Last but not least, relativity is difficult to accommodate in the fiber bundle representation: the symmetry that "gauge" gravity is not the same as those in gauge fields because they are external symmetries and all the symmetries of gauge theories are internal symmetries. Internal symmetries are not associated to coordinate changes.

Moreover, there are no-go results pertaining to show that the Poincaré group P of relativity and the gauge groups G cannot be pasted together in a non-trivial way, i.e. other than$U = G \otimes P$.[11] The no-go results are forbidding us from constructing a non-trivial unified theory of gravitation and gauge fields (Cao 1998),(Maudlin 1996). There is unification in the fiber bundle framework, but it is open to objections raised usually against scientific unification. Do we have a non-trivial quotient space in 6 when gravitation is involved? Maudlin's fiber bundle metaphysics inherits these problems and is bound to address duality within the fiber bundle metaphysics. Duality could hamper its claims of generality and make the metaphysician loathe fiber bundle metaphysics. If fiber bundle unifies indeed all interactions, then spuriousness and trivialness need to be addressed. Both are aggravated for mathematical representations and formalism. R. Healey echoed similar worries when he accepts the unificatory power of the fiber bundle framework "by revealing abstract similarities that may not be apparent at the level of the quite different basic physical laws governing them" (Healey 2007, 44), but warns that fiber bundle does not provide *explanations* in the case of the Aharonov-Bohm effect. The suggestion is that fiber bundle framework is explanatory inert and, arguably impotent in respect of novel predictions. This may illustrate another case

[11] The results of A. Pais, J. Coleman, A. McGlinn from the 1960s are referenced in (Cao 1998, 331).

in which explanation and unification are decoupled (esp. in M. Morrison's sense).[12] Unification and geometrization of the fiber bundle framework need more work on the behalf of the philosophers of physics adopting [1] who may want to analyze all these aspects together with its explanatory and, last but not least, predictive power.

5.3 Realism about internal spaces

Fiber bundle discloses the deep link between internal directions at different spacetime points—*when* a gauge field is present. The structure group G of electromagnetism is U(1) and its principal fiber bundle $\mathbf{P}(M, \mathrm{U}(1))$ is trivial because U(1) is a commutative group. The electromagnetic potential in the fiber bundle formulation is not anymore a field in the spacetime, but a geometrical transformation between bundles. Both in electromagnetism and in Yang-Mills theories the vector bundles are Lie groups and the bundle is a principal bundle.

[16] *Gauge fields are the connection on a fiber bundle*

The "fiber bundle framework" geometrizes gauge fields by identifying them with a connection on a fiber bundle. But [16] may seem *ad-hoc* and perhaps trivial enough. For Healey, taking the gauge field (for example the electromagnetic field) to be *the* connection on a fiber bundle is "more than just a category mistake [...] it is to ignore the element of conventionality involved in choosing one out of a continuum of gauge-equivalent connections, each grounding a different path-dependent notion of color-similarity." (Healey 2008).

There is a suggestion to elude conventionality: aim for more realistic interpretations of fiber bundles. Here is a first proposal, perhaps deemed to be merely terminological. Maudlin does not emphasize enough the importance of *internal space* in gauge theories. Witness that fiber bundle framework is only partially interpreted. Fiber bundle framework associates a whole *internal space* to every point of the manifold, which is not reducible to tangent or dual spaces. Although we speak of internal spaces or internal dimensions, we do not think of them as real internal dimensions or spaces on along which one can "travel" as in the 4D spacetime. It is not clear whether and how we connect the internal and the external degrees of freedom.

An alternative way of talking of bundles is to call them *internal spaces* or *internal dimensions* (Penrose 2005, Ch. 15). Internal spaces are not simply identical to fiber bundles, although the latter are suitable representations of the former. Perhaps talking about internal spaces can assuage Maudlin's argument and head it towards a more realistic interpretation of fiber bundle. One step toward this interpretation is to see how directions in physical space relate to the directions in internal space—and this is not trivial.[13] But with its un-interpreted structures, fiber bundle framework does not clarify this major point: are internal spaces really spaces or they have a different ontological status? One possible way is to follow what modern Kaluza-Klein theories have suggested and consider internal spaces *on par* with spacetime.

[12] The literature on the spuriousness and triviality of unification as well as its relation to explanation mentions cases of spurious unificatory mathematical representations (Morrison 2000),(Kitcher 1976),(Kitcher 1981).

[13] This result is a development of fiber bundle, not discussed in the literature on gauge theories. See (Cao 1998, 325).

There are other non-standard solutions available. Some proposed a different theory of gravitation, the Einstein–Cartan theory, more suitable to be gauged similarly to Yang-Mills theories (Liu 2003). Others look for Lie groups for gravitation: G. Lisi used the group E_8 in a promising Grand Unified Theory on a principal bundle and was able to embed the Standard model into it (Lisi 2007).

For some others (Baez&Hurta), point particles are not enough: we need higher dimensional extended objects (typically, 1-dimensional objects) to replace points and Lie 2-groups instead of the trivial Lie groups. This looks like denying a variant "pointillism" in gauge theories, in the spirit of Butterffield's project (Butterfield 2006). The current work on the so-called "higher energy gauge" as generalizations of gauges for higher-dimensional extended objects is promising (Baez and Huerta 2010).[14]

And, for altogether another crowd, the only way out of bundle representation toward more realistic interpretation of gauge is to postulate super-symmetry and perhaps then go stringy... albeit not necessarily by assuming extra dimensions of spacetime (Cao 1998, 331).

5.4 Locality and holonomies: where is special relativity?

A major step taken toward the understanding of gauge theories, the move from global to local symmetries, appeals to special relativity. This is I claim, another missing piece of Maudlin's puzzle. There is a way out of fiber bundle metaphysics if one gives up locality. For R. Healey, gauge theories invoke an ontology of extended loops that violates locality and implies non-separability. Unlike Maudlin, Healey promotes another mathematical representation of gauge theories, the holonomies.[15] Gauge theories refer to nonlocal properties encoded in holonomies. In Healey's account, local gauge symmetries become formal and with no empirical content. His argument is couched in skepticism about the gauge-dependent terminology. In his debate with Healey, Maudlin labels his metaphysics as "hyper-local": gauge theories are more local than classical physics.[16] The battlefield is the Aharonov-Bohm effect which, in the received view, does not violate locality and separability (Maudlin 1998). Whether the new type of holism advanced by Healey precludes the use of the fiber bundle representation is a different discussion, but the holonomy framework has interesting advantages over the fiber bundle: it accommodates better quantum interactions. Because of their manifest non-locality, one can ask whether holonomies are suitable to a relativistic formulation. The same questions, unaddressed by Maudlin, linger over the fiber bundle metaphysics: in what sense is the replacement of the standard metaphysics compatible with the prescriptions of special relativity? What is the advantage of adopting fiber bundle in respect of quantization and the interaction of quantum particles with fields? Arguably, once we adopt the fiber bundle metaphysics, the definitions of locality, separability, local action has to be modified to account for internal spaces. It's difficult to say whether or not this is a quick maneuver, but the problem has to be addressed. The intuition one might have is the separability, locality, causation and alike, concepts built upon a standard metaphysics, are *not* defined on the total space B, but on the base space M (Healey 1997, 35).

Last but not least, the question about how the fiber bundle metaphysics mesh with relativity can be asked in the context of background independence. Does the

[14] String Theory, Loop Quantum Gravity and Spin Foam Models all adopt this perspective.
[15] A holonomy is a quantity associated with a closed curve.
[16] Personal communication (April 2009).

theory satisfy background independence—as one of the most acclaimed ideals of modern physics? This is a difficult question that any project akin to Maudlin's needs to address (Guay 2008).

6 Conclusions

I sketched several rebuttals to the fiber bundle metaphysics and several answers. I did not endorse here the Lewis-Armstrong theory of universals. My project is more modest: I showed that fiber-bundle metaphysics is open to several objections and tried to address it. Nonetheless, I hope that my arguments have wider consequences. I believe that coexistence and cooperation between metaphysics and sciences is possible. Both camps learn lessons from the "fiber bundle metaphysics" episode. Metaphysicians should "fragment" their area in smaller domains of applicability: they should not expect metaphysical concepts to work "all way down", when representing gauge fields, spacetime and the like. They should adopt an ontological modesty and admit that metaphysics is true for a specific ontological *niche* in which our everyday experience and ordinary language reign. Philosophers of science trying to reform metaphysics, particularly ontology, in the line of [1] should pay more attention to the ontological commitments and interpretations of the scientific theories acting as reformers. "Practice what you preach!" is my advice for the proponent of the new scientism. I have touted here two elements: a preferred interpretation and a certain form of scientific realism are the must. Maudlin failed to provide them so the metaphysician is walk unabashed. In this respect, Maudlin's project is representative, but unfortunately incomplete: its success is yet to come. Also we need to be charitable with the whole enterprise: it is not ordained to answer all questions and to solve all problems of metaphysics and modern physics!

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